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ABSTRACT

The problem of targeting a probe for Mars atmospheric entry from a manned flyby vehicle during planetary approach is investigated. The effect of errors in thrust application both at separation from the manned vehicle and at the midcourse correction is studied. It is concluded that reasonable accuracy can be achieved without excessive propulsion requirements.

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SUBJECT: Analysis of Errors in Targeting an Atmospheric Entry Probe for Impact on Mars - Case 233

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FROM: R. N. Kostoff
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TECHNICAL MEMORANDUM

1.0 INTRODUCTION

In a recent study(1) D. B. James considered the problem of targeting probes for Mars from a manned flyby vehicle during planetary approach. Some general results were derived pertaining to the ΔV requirements for impulsive injection of the probes from the flyby trajectory into a trajectory which leads to the probe's impact on Mars several hours prior to periapsis of the flyby vehicle. The present analysis goes one step further by investigating the errors associated with the successive steps of such an operation. Some of the assumptions underlying this error analysis are as follows:

(a) the probes are separated from the flyby vehicle several days before periapsis and incur an initial error in speed and direction;

(b) the probe trajectory error acquired at separation from the manned vehicle is corrected by a midcourse maneuver.

The sequence of operations is thus divided into two steps. Depending on circumstances such as the expected magnitude of the errors, the required targeting accuracy, and constraints on propulsion, the second step, midcourse correction, may follow initial injection by a short or long time interval, or may be dropped altogether. Under normal conditions, however, midcourse correction will lead impact by a day or so, and the required ΔV will be an order of magnitude smaller than the initial velocity increment applied at separation from the flyby vehicle. Under these circumstances the overall propulsion requirements are very modest, and do not pose a serious weight problem in the case of the light probes considered here, with a payload of 50 - 100 lbs.

2.0 DETERMINATION OF INITIAL VELOCITY CHANGE FOR VERTICAL IMPACT OF AN ATMOSPHERIC PROBE ON MARS

Before proceeding to the analysis of errors for the sequence of operations described in the introduction, the general configuration of planet, flyby vehicle (denoted F/V) and probe (P_r) have to be examined. This is best visualized in the rest frame of the planet (Figure 1).

Let C be the center of the planet, and R the radius of the planet, including its sensible atmosphere. S is the point of separation of P_r from F/V T sec before F/V reaches periapsis at P along the trajectory SP. The distance SC is r_S and the distance PC is r_p. F/V's velocity at S is V_F cm sec^{-1} in the direction SP'. The probe's velocity after (impulsive) injection into its course is denoted v_P. For early separation considered here - the reason for this will become obvious below - $r_S/R \gg 1$; also F/V's hyperbolic excess velocity, V_∞ cm sec^{-1}, is assumed to be high; i.e., $V_\infty^2 \approx 8\mu/R$, where $\mu/R \approx 1.2 \times 10^{11}$ cm^2 sec^{-2} is the gravitational potential at R. ω radians is the bending angle of F/V's trajectory, e its eccentricity, and b cm the impact parameter of F/V (miss distance). For the three latter quantities the following relations hold:

$$b = \left(1 + \frac{2\mu}{r_P V_\infty^2}\right)^{1/2} r_P \qquad (1)$$

$$e^2 = 1 + V_\infty^4 b^2/\mu^2 \qquad (2)$$

$$\omega = 2 \sin^{-1} (1/e) \qquad (3)$$

For a close flyby ($r_P \approx R \approx 3.6 \times 10^8$ cm and $V_\infty \approx 10^6$ cm sec^{-1}) one obtains $b \approx 4.0 \times 10^8$ cm, $e \approx 9.35$, and $\omega \approx 13.4°$. Equations (1) to (3) are derived in any standard text on mechanics, cf. e.g. Goldstein.[(2)]

To have the probe enter the Martian atmosphere vertically at I at a time ΔT sec before F/V reaches P, the probe's velocity has to be changed at separation into v_P. This involves an increase in magnitude ΔV as well as a change in direction, described by the angle α (see Figure 2). Thus, it is required that

$$\int_S^I \frac{ds}{u(s)} = T - \Delta T \tag{4}$$

where

$$T = \int_S^P \frac{ds}{v(s)} \tag{5}$$

The integral in Equation (4) is evaluated along P_r's trajectory, with u(s) denoting its instantaneous velocity; the integral in Equation (5) is evaluated along F/V's trajectory, with v(s) denoting F/V's instantaneous velocity.

It is shown in Appendix A that for $r_S/R >> 1$, and the desired straight collision path,

$$T - \Delta T = (1-\delta_P)\, r_S/V_P \tag{6}$$

where δ_P is a small correction factor, typically on the order of 10^{-3} for early separation ($\sim$ 2 days) and a hyperbolic excess velocity large compared to the escape velocity. The correction ensues both from gravitational acceleration as well as from the finite magnitude of R.

Similarly (cf. Appendix A)

$$T = (1-\delta_F)\ r_S/V_F \tag{7}$$

with δ_F of about the same magnitude as δ_P, provided $r_P \approx R$. Hence,

$$V_P \approx \frac{T}{T-\Delta T}\ V_F$$

$$= \frac{V_F}{1-t} \tag{8}$$

where $t = \Delta T/T$.

It can be seen from Figure 2 that

$$V_P = V_F \cos\phi + \Delta V \cos(\theta-\phi) \tag{9}$$

where (see Figures 1 and 2) ϕ is the probe separation angle which, for vertical entry at I, is given by

$$\sin\phi \approx \phi = \alpha = b/r_S \approx \frac{b}{V_F T} \tag{10}$$

and θ is the thrust angle at probe separation. For T = 2 days and $b = 4.0 \times 10^8$ cm, $\sin\alpha \approx 2.3 \times 10^{-3} \approx \alpha$. Substitution of Equations (9) and (10) into Equation (8) yields a relation between V_F, ΔV, θ, and α. To evaluate ΔV or θ separately a further relation is needed. This is obtained from consideration of angular momentum. To set the probe on a course such that it enters the planet's atmosphere vertically at I, where it has no angular momentum with respect to the center of the planet, an angular momentum $\Delta\ell$ has to be imparted to the probe at S. This is given by

$$\Delta \ell = r_s \, \Delta V \sin(\theta - \phi) = bV_\infty$$

$$= b\left(1 - \frac{2\mu}{r_S V_F^{\,2}}\right)^{1/2} V_F$$

$$\approx b\left(1 - \frac{\mu}{r_S V_F^{\,2}}\right) V_F \qquad (11)$$

Substitution of Equations (10) and (11) into Equation (8), and writing $\chi = \Delta V/V_F$, yields, to second order in α,

$$\frac{1}{1-t} = (1 - \alpha^2/2) + \chi - \frac{\alpha^2}{2\chi} \qquad (12)$$

and to first order

$$\chi = \frac{t}{1-t} \qquad (13)$$

By applying the Law of Sines to Figure 2 and using Equation (13) one obtains

$$\theta \approx \frac{\alpha}{t} \qquad (14)$$

For $\alpha \approx 2.3 \times 10^{-3}$ and $t = 1/8$ (6 hours lead time), $\chi = 1/7$ and $\theta \approx 2 \times 10^{-2} \approx 1°$. Consequently, the initial velocity increment of P_r is predominantly tangent to the flight path of P_r.

3.0 ANALYSIS OF ENTRY ANGLE ERRORS ACQUIRED AT SEPARATION

It is now possible to proceed to the entry angle error analysis for the initial thrust application to the probe. One may distinguish three main sources of error: (1) an error in the magnitude of ΔV, to be denoted $\delta\Delta V$, (2) an error in thrust angle $\delta\theta$, (3) an error δr_S in the measured distance of S from C.* If the entry angle is denoted by γ, the total average initial error squared, $\delta\gamma_i^2$, may be written as:

$$\delta\gamma_i^2 = \delta\gamma_{i1}^2 + \delta\gamma_{i2}^2 + \delta\gamma_{i3}^2 \tag{15}$$

P_r's entry angle into the atmosphere is obtained from conservation of angular momentum. Let primed symbols denote the actual values of P_r's parameters of motion resulting from thrust application at S(cf. Figure 3). For the desired vertical entry, the entry angle, γ, is zero. Also, the aforementioned error sources are independent. Thus each of the three error components, $\delta\gamma_{ij}$ (j = 1, 2, 3) is given by:

$$\delta\gamma_{ij} = \sin^{-1}\left[\frac{r_S\ V_P'\ \sin\ \delta\phi}{R\ V'(R)}\right]_j$$

---

*If the flyby mission were performed today one would have to add an additional error, due to uncertainty in the flight path angle α, or an equivalent error in the estimate of b. Post-flight analysis of the Mariner IV mission reveals an error of ∿ 1000 km in the estimate of b. This error was constant over most of the interplanetary cruise, when the spacecraft moved in the heliocentric field; after the vehicle had moved for a considerable time in the gravitational field of Mars the error started to decrease, and became negligible near encounter of the planet. Consequently, the bulk of the error was ascribed to the uncertainty in the magnitude of the astronomical unit, which is of the same order of magnitude. If current predictions[4] on the reduction of this uncertainty to ∿ ±10 km are correct, and with improved tracking and navigation methods envisioned for the mid-70's,[4] the error in α loses much of its significance.

$$= \sin^{-1}\left[\frac{r_S \sin \delta\phi}{R\left(1 + \frac{2\mu}{R\, V_P'^2} - \frac{2\mu}{r_S\, V_P'^2}\right)^{1/2}}\right]_j$$

$$\approx \sin^{-1}\left[\frac{r_S}{b} \sin \delta\phi\right]_j \approx \sin^{-1}\left[\sin \delta\phi/\alpha\right]_j \qquad (16)$$

Here $V'(R)$ is the actual velocity of the probe at R, $\delta\phi$ is the angle between V_P' and SC, i.e., $\delta\phi = \phi' - \alpha$; the subscript j refers to one of the three error sources mentioned above, which determines the values of the terms in the square brackets. The three error terms in Equation (15) will now be examined separately.

(a) $\delta\gamma_{i1}$

This error results from an error in the magnitude of the velocity increment imparted to P_r at separation, with the thrust angle θ assumed given by Equation (11). In this case

$$\Delta V' = (1+\varepsilon)\Delta V \qquad (17)$$

This relation holds also for any two components into which $\Delta V'$ may be resolved. Reference to Figure 4 and use of Equations (13) and (14) shows that

$$\sin \delta\phi = \sin(\phi' - \alpha) \approx \tan \phi' - \tan \alpha$$
$$\approx (1-t)\varepsilon\alpha \qquad (18\text{ a})$$

and

$$V_P' \approx (1+\varepsilon t)V_P \qquad (18\text{ b})$$

which justifies substitution of b for $R\left(1+\frac{2\mu}{RV_P'^2} - \frac{2\mu}{r_S V_P'^2}\right)^{1/2}$.

Substitution of Equation (18 a) into Equation (16) yields

$$\delta\gamma_{i1} \approx \varepsilon \text{ radians} \tag{19}$$

For $\varepsilon \approx 3 \times 10^{-2}$, a plausible estimate with any sizeable ΔV, this error is, therefore, quite insignificant.

(b) $\delta\gamma_{i2}$

In the evaluation of $\delta\gamma_{i2}$, ΔV is considered nominal, while $\theta' = \theta + \delta\theta$. $\delta\theta$ is estimated to be on the order of 10^{-2} radians.(3)(4) Reference to Figure 5 shows that

$$\begin{aligned} \sin\delta\phi &\approx \sin\phi'' - \sin\alpha \\ &\approx \tan\phi'' - \tan\alpha \\ &\approx t\,\delta\theta \end{aligned} \tag{20}$$

$$\begin{aligned} V_P'' &\approx [1 + O(t\theta\delta\theta)]V_P \\ &\approx V_P \end{aligned} \tag{21}$$

Hence

$$\delta\gamma_{i2} \approx \sin^{-1}[t\delta\theta/\alpha] \tag{22}$$

With $\alpha \approx 2.3 \times 10^{-3}$, $t = 1/8$ and $\delta\theta = 10^{-2}$ Equation (22) yields

$$\begin{aligned} \delta\gamma_{i2} &\approx \sin^{-1}(0.545) \\ &\approx 33^\circ \end{aligned}$$

It appears, therefore, that relatively small errors in thrust angle result in large errors in entry angle. It will presently be seen that this is the dominating error resulting from the thrust application at separation. It should also be noted with respect to Equation (22) that the maximum possible argument of $\sin^{-1}$ is 1. This value corresponds to grazing incidence on the atmosphere. Whenever the value of unity is exceeded, a complete miss of the atmosphere is indicated.

(c) $\delta\gamma_{13}$

This error in entry angle ensues from the uncertainty in the measurement of r_S. Its effect on the entry angle, though small, is somewhat complex: the wrong estimate of r_S is equivalent to an error in T, which leads to an error in the thrust angle, which in turn causes an error $\delta\phi$ in the separation angle. In the Appendix, Equation (A-11), it is shown that

$$\delta\phi \approx \alpha \frac{\delta r_S}{r_S} \qquad (23)$$

Thus, by substitution of Equation (23) into Equation (16),

$$\delta\gamma_{13} \approx \frac{\delta r_S}{r_S} \qquad (24)$$

Even under the assumption that $\delta r_S \sim 10^{-3}\, r_S$,[4] $\delta\gamma_{13} \approx 10^{-3}$, which is small compared to the other errors ensuing from the initial thrust application.

4.0 THE MIDCOURSE MANEUVER

For the drag probe considered here the major constraint is the magnitude of entry angle into the atmosphere. Various considerations prescribe that entry be close to vertical.(7) Thus, in view of the trajectory error, a midcourse correction may be necessary to reduce the departure from vertical entry to within acceptable limits. This constraint on entry angle may then be expressed as

$$\gamma \leq \gamma_0 \tag{25}$$

A plausible value for γ_0 is 30°. The errors associated with the midcourse maneuver will be evaluated in the following analysis.

Let the midcourse correction be applied at the point M on the trajectory (between S and I'), at a time T_m before periapsis of F/V (see Figure 6). The distance MC is denoted r_m. T_m should be sufficiently large compared to ΔT, otherwise, P_r's trajectory would have to be bent sharply, involving an excessive velocity increment. This would, indeed, be necessary if the trajectory change were postponed until just prior to P_r's entry into the planet's atmosphere at I'. Consequently, $r_m >> R$.

The main purpose of the midcourse maneuver is, therefore, to assure P_r's vertical entry into the planet's atmosphere at I_C (cf. Figure 6); from M it is no longer possible to aim P_r for vertical entry at I. The most economic way of changing the direction of P_r's trajectory at M, without significantly changing P_r's total velocity, is to impart to P_r a velocity increment Δu normal to its direction of flight.* Δu is then determined by conservation of angular momentum, i.e.,

$$r_m \Delta u \approx R \sin \delta\gamma_1 \, V''(R) \tag{26}$$

In view of the previously calculated relative magnitude of the different components of $\delta\gamma_1$, only $\delta\gamma_{12}$ needs to be considered. Combination of Equations (16), (20) and (26) yields

---

*For large r_m the angle between the direction of flight and the radius vector is negligible if the thrust angle is large.

$$\Delta u = \frac{r_S V_P'' \, \delta\phi}{r_m}$$

$$\simeq \frac{r_S V_P'' t}{r_m} \, \delta\theta$$

$$= \frac{t \;\delta\theta}{(\tau - t)} \; V_F \qquad (27)$$

where $\tau = T_m/T$

Equation (27) bears out the statement made above with regard to the relative magnitude of T_m and ΔT. A fuller discussion of this point will, however, follow the forthcoming analysis of errors associated with the midcourse maneuver.

The error $\delta\gamma_m$ resulting from uncertainties in the application of thrust at M will now be resolved into its separate components. Each component $\delta\gamma_{mj}$ may be expressed as

$$\delta\gamma_{mj} = \sin^{-1} \left[\frac{\delta(r_m \;\Delta u)}{R \; V''(R)}\right]_j \qquad (28)$$

where the subscript j (1, 2, 3) denotes the jth source of error.

The following relations are then obtained:

(a) $\delta\gamma_{m1}$

This is the error resulting from an uncertainty in the magnitude of Δu. With

$$\Delta u' = (1 + \varepsilon_m)\Delta u \qquad (29)$$

Substitution in Equation (29) yields

$$\delta\gamma_{m1} \approx \sin^{-1}\left[\frac{r_m\ \varepsilon_m\ \Delta u}{R\ V''(R)}\right]$$

$$\approx \sin^{-1}\left[\frac{\varepsilon_m\ r_S V_P''\ \delta\phi}{R\ V''(R)}\right]$$

$$\approx \sin^{-1}\left[\frac{\varepsilon_m t\delta\theta}{\alpha}\right] \qquad (30)$$

If it is assumed that ε_m is constant, then to a first approximation $\delta\gamma_{mi}$ is independent of T_m. With $\varepsilon_m \approx 5 \times 10^{-2}$, and with the previously used values of the other parameters

$$\delta\gamma_{m1} \approx 3 \times 10^{-2} \qquad (31)$$

Constancy of ε_m may not be too realistic; another assumption would be constancy of $\varepsilon_m \Delta u = \delta\Delta u$. In that case for $r_m/R >> 1$

$$\delta\gamma_{m1} \approx \sin^{-1}\left[\frac{T(\tau-t)}{R}\ \delta\Delta u\right] \qquad (32)$$

With the sample parameter values used previously, and assuming $\tau = 1/4$

$$\delta\gamma_{mi} = 5.4 \times 10^{-2} = 3° \qquad (33)$$

Thus, the error in thrust magnitude has no significant effect on entry angle error.

(b) $\delta\gamma_{m2}$

This error is a result of the uncertainty in thrust angle θ_m. It is easy to show, with the aid of Figure 7, that

$$\delta\Delta u = \Delta u' - \Delta u \simeq (1-\cos\delta\theta_m)\Delta u$$

$$\simeq \frac{\delta\theta_m^{\ 2}}{2}\ \Delta u \qquad (34)$$

Inserting previously assumed parameters, and assuming $\delta\theta_m = 5 \times 10^{-2}$, gives

$$\delta\gamma_{m2} \simeq 1.2 \times 10^{-4} \qquad (35)$$

(c) $\delta\gamma_{m3}$

This is the error arising from the uncertainty in r_m, δr_m. Consequently

$$\delta\gamma_{m3} \simeq \frac{\delta r_m t\,\delta\theta V_F}{(\tau-t)R\ V''(R)}$$

$$\simeq \frac{\delta r_m t\,\delta\theta}{R(\tau-t)} \qquad (36)$$

Insertion of the selected parameter values and $\delta r_m = R$ yields, for this error

$$\delta\gamma_{m3} \simeq 10^{-3} \qquad (37)$$

5.0 CONCLUSIONS

The analysis of errors carried out in the foregoing sections reveals that, under reasonable assumptions about the time of separation and the time of the midcourse maneuver, two dominant uncertainties in targeting arise from errors in thrust application at these two points.

In separation at large distance from the planet the impulse required to launch the probe for early arrival at the planet has to be applied at a very small angle to the trajectory of the flyby vehicle. Consequently, in view of the large distance, the small initial error in angle is magnified to the large error in entry angle at the planet given by Equation (21).

The principal error source in the midcourse maneuver, on the other hand, is the uncertainty in the magnitude of thrust. This is due to the fact that at this point the impulse is applied approximately normal to the probe's trajectory, such that the magnitude of the impulse determines the deflection of the probe from its original path. This error is given by Equation (32).

Conversely, by setting an upper limit on this error, say γ_o, which is the final error, an upper limit can be obtained for $\tau = T_m/T$ in terms of γ_o and T, and τ can be eliminated in the considerations leading to the selection of a proper value for the separation time T.

A reasonable procedure for selection of T, to be described in the following, starts with the total relative velocity increment, $\Delta V_T/V_F$. This is the sum of $\Delta V/V_F$ at separation, Equation (13), and $\Delta u/V_F$ at midcourse, Equation (27). Thus

$$\frac{\Delta V_t}{V_F} = \frac{t\ \delta\theta}{\tau - t} + \frac{t}{1-t}$$

$$= \frac{\Delta T\ \ \delta\Delta u\ \ \delta\theta}{R \sin \gamma_o} + \frac{\Delta T}{T-\Delta T} \qquad (38)$$

according to Equation (32).

Values for all quantities on the right side of this equation are then arbitrarily assumed. ΔT and γ_o are not left as variables because first, a given mininum ΔT is prescribed by the mission requirements, and second, as mentioned before, an upper bound must be set on the entry angle for sensible interpretation of the measurements.(7) Reasonable values for the fixed parameters in Equation (38) are:

$$\Delta T = 4.0 \times 10^4 \text{ sec} \approx 1/2 \text{ day}$$

$$\gamma_o = 2.5 \times 10^{-1} \text{ radians}$$

$$\delta \Delta u = 10 \text{ m/sec}$$

$$R = 4 \times 10^6 \text{ m}$$

$$\delta\theta = 10^{-2} \text{ radians}$$

Figure 8 shows the graph of $\Delta V_T/V_F$ versus $T/\Delta T$. As T gets very large $\Delta V_T/V_F$ approaches $\sim 4.0 \times 10^{-3}$ asymptotically, while as T approaches ΔT, $\Delta V_T/V_F$ becomes infinite. The portion of the curve for large $T/\Delta T$ should, however, not be taken literally. Obviously, at excessive planetocentric distances, corresponding to large T, the errors in thrust angle and ΔV exceed the nominal values of these quantities, this setting effective early limits for probe launch. Under these circumstances additional factors may also have to be considered, such as acceptable limits on early arrival time.

Figure 8 also shows the overall weight growth factor of the probe versus $T/\Delta T$. This is defined as the ratio of the total weight of the probe system (including its propulsion system), to the probe payload weight at entry into the planet's atmosphere. The payload weight is the sum of the probe's subsystems and structure. The growth factor is determined both by the total velocity increment required in the mission profile, including initial injection and the midcourse maneuver, as well as by the parameters of the propulsion system employed. With the present state of the art, the requirements of the probe mission profile, considered here, appear to be best satisfied by liquid fuel engines with relatively high I_{sp}, > 300 sec, but with a rather low mass fraction, λ, of ~ 0.7. The curve for the growth factor presented in Figure 8 is based on these parameter values and a flyby velocity $v_F = 10$ km sec^{-1}.

Figure 8 reveals that the growth factor increases rather mildly from large $T/\Delta T$ to $T/\Delta T \sim 8$, below which value it starts to rise increasingly fast. If the growth factor is to be held below 2, T should be greater than ~ 4 days, with the value of ΔT selected above. For such a value of T the above analysis can still be assumed valid. Also, doubling the gross weight of a light payload of 50 - 100 lbs cannot be considered an excessive burden in the framework of the 1975 manned Martian flyby mission. It may be safely concluded from the results of this analysis that for $\Delta T \sim 1/2$ day, a realistic separation time is on the order of five days.

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Attachment
Figures 1-8

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APPENDIX A

A. Time of Flight of the Manned Module and the Probe

From energy conservation of the probe, valid until entry into the atmosphere, one obtains for its instantaneous velocity at a distance r from the planet

$$V(r) = V(r_S)\ [1 + \frac{2\mu}{r_S V^2(r_S)}\ (\frac{r_S}{r} - 1)]^{1/2}$$

$$= V_P[1 + \beta(r_S/r-1)]^{1/2} \qquad \text{(A1)}$$

with

$$\beta = \frac{2\mu}{r_S V_P^2} << 1 \qquad \text{and } V_P \equiv V(r_S)$$

For $r_S = 1.7 \times 10^6$ km (separation $\sim$ 2 days prior to periapsis) and $V_P = 10$ km sec^{-1}, $\beta = 5 \times 10^{-4}$.

The time of flight along a straight course from S to I (cf. Fig. 1) is then

$$\tau = -\int_{r_S}^{R} \frac{dr}{V_P[1+\beta(r_s/r-1)]^{1/2}}$$

$$= \frac{r_S}{V_P} \int_1^{u_1} \frac{du}{u^2[\alpha+\beta u]^{1/2}} \qquad \text{(A3)}$$

where $u = r_S/r$, $u_1 = r_S/R >> 1$ and $\alpha = 1-\beta$. Thus

$$\tau = \frac{r_S}{V_P}\left\{ - \frac{(\alpha+\beta u)^{1/2}}{\alpha u} \Bigg|_1^{u_1} - \frac{\beta}{2\alpha^{3/2}} \log \frac{(\alpha+\beta u)^{1/2}-\alpha^{1/2}}{(\alpha+\beta u)^{1/2}+\alpha^{1/2}} \Bigg|_1^{u_1} \right\}$$

$$= \frac{r_S}{V_P}\left\{1 - u_1^{-1} - \frac{1}{2}\beta \log u_1\right\} \tag{A4}$$

to first order in the small quantities β and u_1^{-1}.

Thus for the values given above ($u_1 = 5 \times 10^2$)

$$\tau = (1-\delta_P) r_S/V_P \tag{A5}$$

where the correction factor is

$$\delta_P = 3.5 \times 10^{-3} \tag{A6}$$

Similarly T, the time of F/V's flight along its hyperbolic trajectory from S to P, is given by

$$T = -\int_{r_S}^{r_P} \frac{dr}{[V_\infty^2 + 2\mu/r - \ell^2/r^2]^{1/2}} \tag{A7}$$

where ℓ is the specific angular momentum of F/V given by

$$\begin{aligned} \ell^2 &= V^2(r_P) r_P^2 \\ &= V_\infty^2 r_P^2 + 2\mu r_P \end{aligned} \tag{A8}$$

Let:

$$\beta' = \frac{2\mu}{r_S V_\infty^2} = 5 \times 10^{-4} \tag{A9.1}$$

$$u = r_S/r \; ; \; u_P = r_S/r_P = 5 \times 10^2 \tag{A9.2}$$

$$\gamma = -1/u_P^2 - \beta/u_P \tag{A9.3}$$

Then

$$T = r_S/V_\infty \int_1^{u_P} \frac{du}{u^2(1+\beta' u+\gamma u^2)^{1/2}}$$

$$= r_S/V_\infty \left\{ -(1+\beta' u+\gamma u^2)^{1/2}/u \Big|_1^{u_P} \right.$$

$$\left. + \frac{\beta'}{4u} [2(1+\beta' u+\gamma u^2)^{1/2} + 2 + \beta' u] \Big|_1^{u_P} \right\}$$

$$= \left\{ 1 + \beta - \beta/2 \log(2u_P) \right\} r_S/V_F$$

$$= \left\{ 1 - \delta_F \right\} r_S/V_F \tag{A 10}$$

with

$$\delta_F = -\beta + \beta/2 \log (2u_P)$$

$$= 1.5 \times 10^{-3}$$

for the assumed conditions. For earlier separation (larger T) the correction factor δ_F becomes correspondingly smaller, since the fraction of total travel time spent within the sensible gravity field of the planet becomes less.

B. Effect of an Error in the Estimate of r_S

From Equation (14) one finds that

$$\theta \simeq \frac{\alpha}{t} \simeq \frac{\alpha r_S}{\Delta T \; V_F}$$

such that

$$\delta\theta \simeq \frac{\alpha\delta r_S}{\Delta T \; V_F}$$

But

$$\phi \simeq \frac{\Delta V\theta}{V_F+\Delta V}$$

whence

$$\delta\phi \simeq \frac{\partial\phi}{\partial\theta}\,\delta\theta$$

$$\simeq \frac{\Delta V\delta\theta}{V_F+\Delta V}$$

$$\delta\phi \simeq \alpha\,\frac{\delta r_S}{r_S} \qquad \text{(A 11)}$$

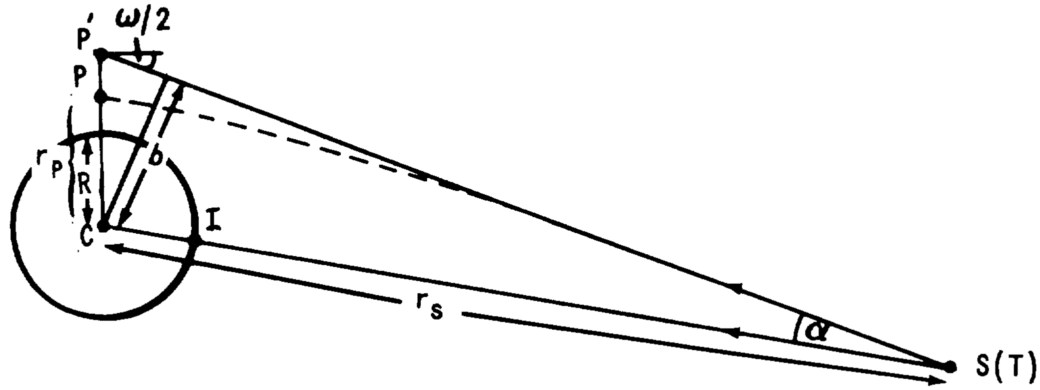

FIG. I. CONFIGURATION OF MARS, FLBY VEHICLE AND PROBE.

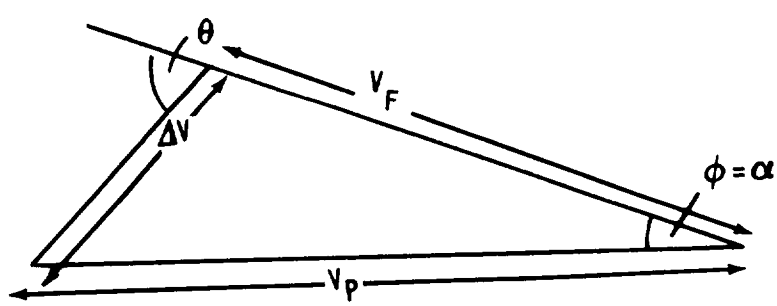

FIG. 2. VELOCITY INCREMENT ΔV AND THRUSTING ANGLE.

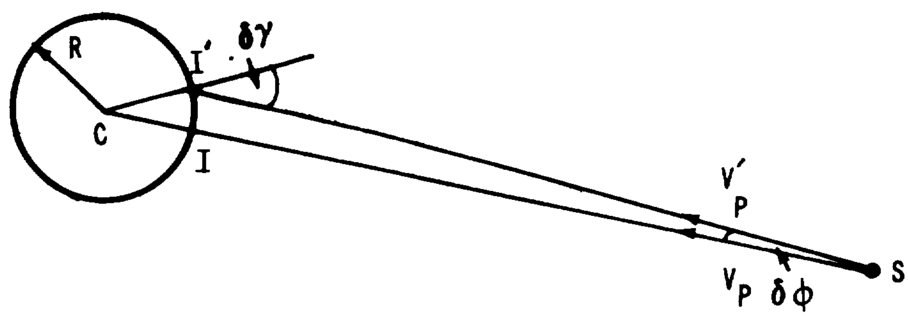

FIG. 3. NOMINAL TRAJECTORY AND ACTUAL TRAJECTORY

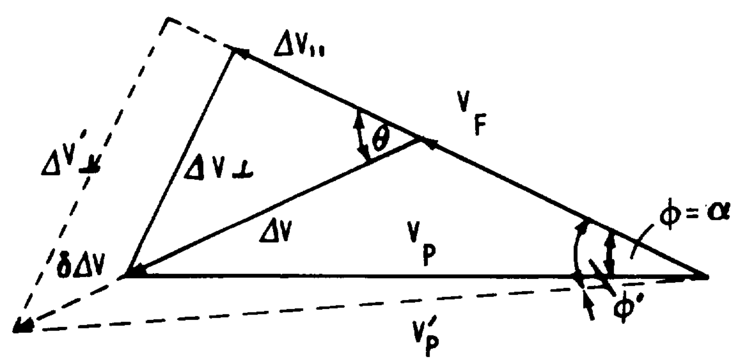

FIG. 4. EFFECT OF ERROR IN MAGNITUDE OF VELOCITY INCREMENT ON PROBE VELOCITY AND SEPARATION ANGLE

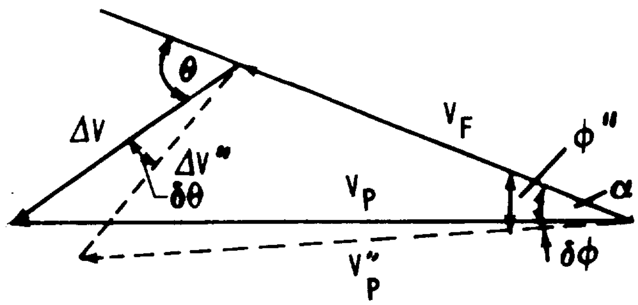

FIG. 5. EFFECT OF THRUST ANGLE ERROR ON PROBE VELOCITY AND ENTRY ANGLE.

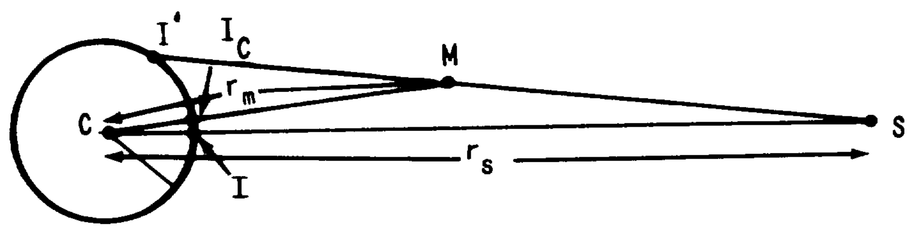

FIG. 6. CONFIGURATION OF MIDCOURSE MANEUVER

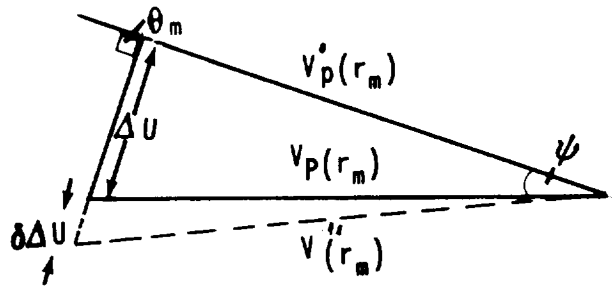

FIG. 7. EFFECT OF ERROR IN MAGNITUDE OF Δ U ON THE DIRECTION OF Pr.

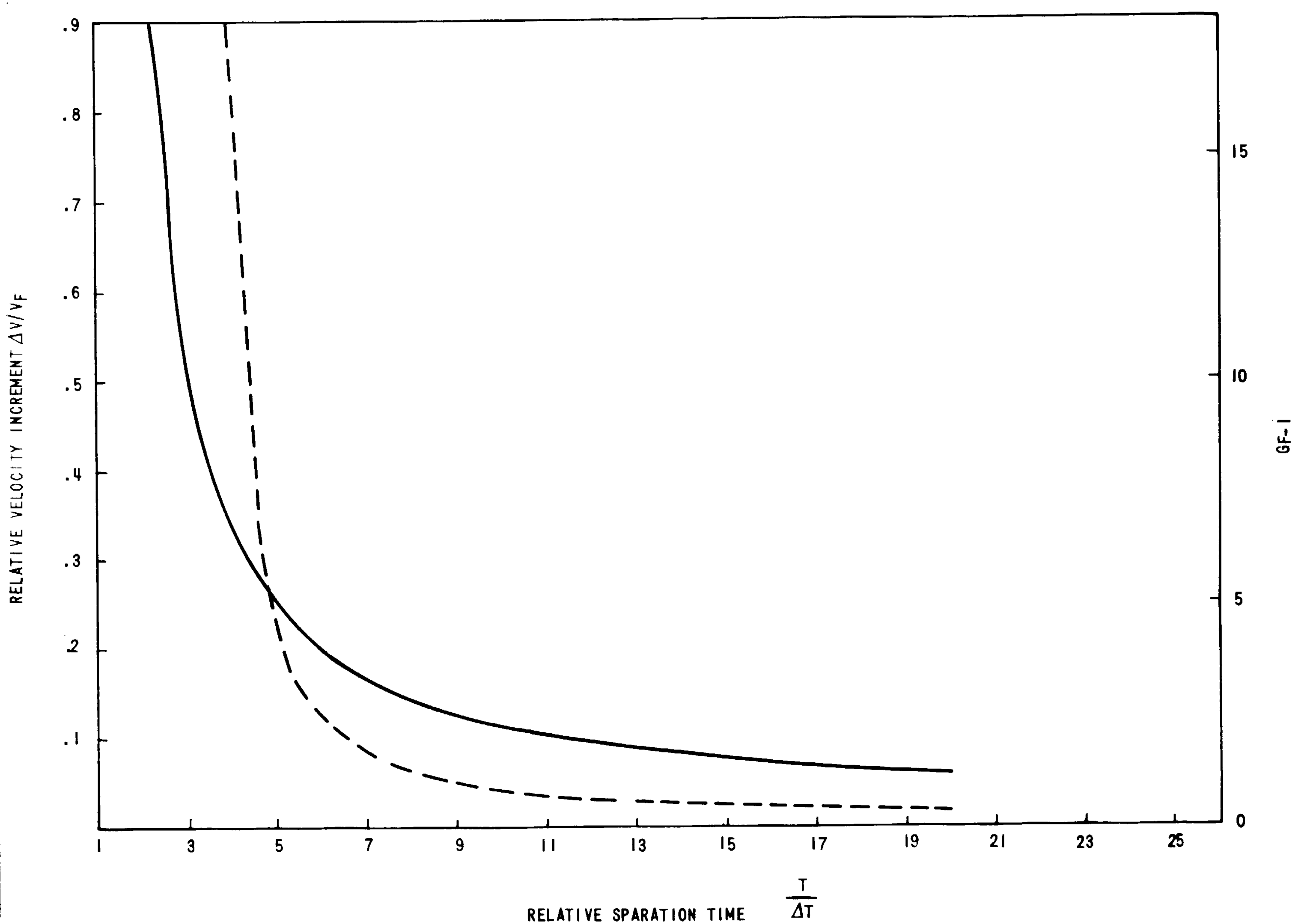

FIG. 8. VELOCITY INCREMENT (SOLID LINE) AND GROWTH FACTOR (DASHED LINE) VS SEPARATION TIME.